# Hardware-Aware Static Optimization of Hyperdimensional Computations

Pu (Luke) Yi and Sara Achour





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• Improved performance and energy consumption



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- Improved performance and energy consumption
- Higher storage density and faster memory access times
- Dense 3D interconnect; significantly higher bandwidths







These emerging technologies are highly promising, but are less reliable than conventional memory/compute substrates and occasionally corrupt bits.

**Why does this occur?** Conformational changes in materials, static errors from immature fabrication processes, sensitivities to the environment.



#### Performing Computation on Emerging Hardware Technologies

Performing classical computation on these hardware substrates is challenging because *where* corruptions occur in the program & the program data has a substantial impact on the computed result.



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# Over the years a number of error mitigation techniques have been developed to work around the problem.

*E.g., error-correcting codes, precise/approximate data partitioning, redundant computation.* 

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These mitigations introduce hardware and software overheads that affect projected energy/performance improvements.

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### What else can be done? We can change *how* we perform computation.

This work focuses on *hyperdimensional computation*, a computational model that is naturally resilient to error.

**Hyperdimensional computing (HDC)** is a highly error-resilient novel computational paradigm that originated from the cognitive science community.

#### $[0,1,1,1,0,\ldots,0,1,0]$

The basic unit of information is a *hypervector*, a high-dimensional binary vector.<sup>1</sup>

1. This talk overviews Binary Spatter Code (BSC), a variant of HDC that works with dense binary hypervectors.

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$$[0,1,0,1,0,\ldots,0,0,0]$$

#### **Doesn't matter where a bit corruption occurs!**

In HD computing, information is encoded in the hamming distances between hypervectors.

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$$[0,1,0,1,0,\ldots,0,0,0]$$
  
 $[0,1,1,1,0,\ldots,0,1,0]$ 

Many bit corruptions are required to make a non-negligible change in the distance. **Highly error resilient!** 

### Hyperdimensional Computation

Hyperdimensional Computation (HDC) can perform a variety of tasks

- *Data structures* construction and querying of database, graph, tree, finite automata, etc.
- <u>Processing tasks</u> information retrieval, load balancing, analogical reasoning
- *Machine learning* time-series data classification/edge/low-power

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Heim's analysis is *static*, and completes in milliseconds at compile-time.

We present *Heim*, the first static analysis-based optimizer for optimizing hyperdimensional computations to execute with acceptable accuracy on emerging hardware technologies.

Heim is **hardware-aware** and optimizes computations to execute accurately on emerging hardware technologies.

Buffer								
	H	CAM	CAM		CAM	••••	MLs CAM Cell	e Sense Block
larger		CAM Cell	CAM Coll		CAM Cell		CAM	Bloc
Prech		CAM	CAM		CAM		CAM	Sense Block
		CAM	CAM		CAM Cell		CAM	Sense Block





### Hyperdimensional Computation by Example

# **Knowledge Graph Data structure.** directed graph with labeled edges and nodes.

Node Label = "concept"

Edge Label = "relation"

Edge Direction = "interaction"



Student knowledge graph

#### Edge Queries. Ask about relationships between nodes, or concepts.

**Query.** How many students like apples?

#### <mark>Result.</mark> Two students

# of students with "likes" relations that point to the Apple concept.



We want to execute the "apples" query on a piece of hardware that stores information in an **Two-Bit-Per-Cell Resistive RAM (RRAM) storage array**, an information-dense emerging memory technology that is prone to error.





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# First, we need to construct the *atomic elements* of knowledge graph:

Relations = {likes,plays}

Concepts = { jack, mary, apple, tennis, banana}

Interactions= {act,target}



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How do we do this?

We generate a random binary vector, or atomic hypervector, for each type of node label (concept), edge label (relation), and edge direction in the knowledge graph.



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apple

plays

- Hamming distance (HD) between atomic hypervectors is large!
  - Conceptually, makes sense. The "apples" node and "plays" are not related at all.

[1,0,0,1,1,....,1,0,0]



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We will be building the data structure from the bottom-up.

 $\mathsf{Edges} \to \mathsf{edge \ sets} \to \mathsf{graph}$ 



Now we're ready to encode the data structure as a hypervector using HD computing.

**How do we encode information?** We will compute over the atomic hypervectors!



We want to construct each labeled, directed edge relative to a particular node

<target,likes,jack> points to Mary node

Concepts= { jack, mary, apple, tennis, banana}

**Relations= {likes,plays}** 



#### <target,likes,jack> points to Mary node

```
hv1 = target \circ likes \circ jack
binding operation
XOR
```

For each node, we construct each graph edge by binding together the interaction, relation, and concept hypervectors.

Concepts= { jack, mary, apple, tennis, banana}

**Relations= {likes,plays}** 



#### <target,likes,jack> points to Mary node

hv1 = target  $\circ$  likes  $\circ$  jack

**Binding** creates a hypervector that is dissimilar to the input hypervecvectors HD(hv1,target) is large HD(hv1,likes) is large HD(hv1,jack) is large


hv1 = target o likes o jack <target,likes,jack>

We construct a edge hypervector for each edge that is connected to the mary node.



hv1 = target  $\circ$  likes  $\circ$  jack<target,likes,jack>hv2 = act  $\circ$  likes  $\circ$  apple<act,likes,apple>

We construct a edge hypervector for each edge that is connected to the mary node.



hv1 = target o likes o jack<target,likes,jack>hv2 = act o likes o apple<act,likes,apple>hv3 = act o plays o tennis<act,plays,tennis>

We construct a edge hypervector for each edge that is connected to the mary node.

**Relations= {likes,plays}** 

We next want to create a set of edges that are connected to the mary node.

{ <target,likes,jack>, <act,likes,apple>, <act,plays,tennis> }



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```
hv_mary = hv1 + hv2 + hv3
```

To accomplish this, we bundle the edge hypervectors together



**Relations= {likes,plays}** 

We next want to create a set of edges that are connected to the mary node.

{ <target,likes,jack>, <act,likes,apple>, <act,plays,tennis> }

hv\_mary = hv1 + hv2 + hv3

Bundling creates a hypervector similar to the input hypervectors.

HD(hv\_mary,hv1) is small HD(hv\_mary,hv2) is small HD(hv\_mary,hv3) is small



**Relations= {likes,plays}** 

We next want to create a set of edges that are connected to the mary node.

{ <target,likes,jack>, <act,likes,apple>, <act,plays,tennis> }

hv\_mary = hv1 + hv2 + hv3

We can use the hamming distance to query if an edge belongs to an edge set!

HD(hv\_mary, act  $\circ$  likes  $\circ$  apple) -> small, in set HD(hv\_mary, act  $\circ$  likes  $\circ$  apple) -> large, NOT in set





Next, we build a edge set hypervector for each node in the graph.

hv\_mary = hv1 + hv2 + hv3

hv\_jack = hv4 + hv5 + hv6 + hv7

hv\_tennis = hv8 + hv9

hv\_banana = hv10

hv\_apple = hv11 + hv12

#### Interactions= {act,target}

Concepts= { jack, mary, apple, tennis, banana}

**Relations= {likes,plays}** 



Now we can query for edges. Let's test for the "likes-apples edge": <act,likes,apple> act  $\circ$  likes  $\circ$  apple hv mary = hv1 + hv2 + hv3hv\_jack = hv4 + hv5 + hv6 + hv7 hv tennis = hv8 + hv9hv\_banana = hv10 hv apple = hv11 + hv12

Relations= {likes,plays}



We then perform test for an edge by computing the hamming distance between query edge and each edge set.

HD(hv\_mary, act o likes o apple)

HD(hv\_jack, act o likes o apple)

HD(hv\_tennis, act o likes o apple)

HD(hv\_banana, act o likes o apple)

HD(hv\_apple, act o likes o apple)

**Relations= {likes,plays}** 



If the hamming distance is small, the edge is contained within the node's edge set.

HD(hv\_mary, act  $\circ$  likes  $\circ$  apple) ->

HD(hv\_jack, act o likes o apple)

HD(hv\_tennis, act  $\circ$  likes  $\circ$  apple)

HD(hv\_banana, act o likes o apple)

HD(hv\_apple, act o likes o apple)

- -> small distance, in set!
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#### So, there is some missing information..

#### What distance threshold should we use? How do we

distinguish between a small and large distance?



#### So, there is some missing information..

What distance threshold should we use? How do we distinguish between a small and large distance?

How big are these hypervectors exactly?



#### How are the thresholds and size set currently?

Currently, practitioners dynamically tune the parameters with Monte Carlo simulations

Lack of accuracy guarantees May not generalize well Computationally intensive



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Currently, practitioners dynamically tune the parameters with Monte Carlo simulations

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We present Heim, a static optimizer that analytically derives the size and distance thresholds for an HD computation.



## Static parameter optimization with Heim



#### Heim program specification

```
spec {
    abs-data query = prod(interactions,relations,concepts);
    abs-data ds = sum(4,prod(interactions,relations,concepts));
    thr-query(query, ds, 1, 0.99, 0.01, 0.01);
}
```

Knowledge graph specification

Heim works with a program specification that describes the space of HD data structures to analyze and the desired query accuracies.



Heim chooses a hypervector size and a set of distance thresholds that satisfies all query accuracy constraints in the specification.



**Heim's accuracy guarantee:** on expectation, the accuracy of each described query will converge to the accuracy specified by the most restrictive accuracy constraint.



Heim ensures this *accuracy guarantee* holds over all queries and data structures described in the Heim specification.



Heim accepts a hardware specification which specifies the bit error rates of different storage and compute elements in the hardware architecture. Heim's accuracy guarantees hold in the presence of hardware error.



Heim analytically derives the optimal distance thresholds and the minimum hypervector size required to meet the target accuracy constraints on the target hardware.



Heim's implementation consists of an analytical model of hypervector-query hamming distances for the given data structure, parametrized over hypervector size.



..an accuracy analysis that derives query accuracies from the analytical model.



..and an optimization algorithm that uses the accuracy analysis to find the smallest hypervector size that delivers the desired accuracy.



distribution of hamming distances for matching and non-matching queries.





We observe distance distributions because there is non-determinism introduced by atomic hypervector sampling and hardware error.





variance of the matching/non-matching queries, parametrized over hypervector size.

distance(query,DS)



model.

distance(query,DS)



distance(query,DS)

## Summary of Theoretical Formulations

Formulation	reference	description
WTA-acc, $w=1$ (6)	[Frady et al. 2018]	WTA accuracy for exactly one winner $w=1$
WTA-acc (10)	Section 5.4	WTA accuracy for more than one winner $w > 1$
WTA- <i>prob</i> (12)	Section 5.4	probability of the <i>w</i> positives being in top <i>t</i> .
QDS I (14)	[Kanerva et al. 1997]	single-element sum-of-product set membership
QDS II (15)	[Kleyko et al. 2016]	subset sum-of-product set membership
QDS III (17)	Section 6.6	single-element product-of-sum set membership
Hardware Error (20)	Section 6.8	incorporation of hardware error

To develop this analysis, we apply results from the theoretical cognitive science community and contribute new derivations.

# How does Heim perform?

#### Sound parameter optimization with Heim

We evaluated Heim on five hyperdimensional computing-based data structures, parametrized with five different sizes.

Benchmark	benchmark	query type		data structure and query sizes					
	set	threshold	50k-100k element sets, 1 element/query						
data structures	db-match	threshold		5k-10k fields/record, 50-100 records, m fields/query					
1	kgraph	threshold	1-100 <i>k</i> edges/concept, 100 <i>k</i> +10 concepts, 800 <i>k</i> -1000 <i>k</i> edges,						
	1000-11 (1000-11			1 edge/query, 2 relations					
	nfa	threshold	threshold recognizes str with length $k$ , 1- $k$ character strings/query, 26 letters/alp						
		<i>query</i> : matc	query: matches are substrings of str, non-matches are partial substrings of str						
	db-analogy	winner-take-all (WTA) $k/2-k$ fields/record, $50m-100m$ records, 1 analogy query				50 <i>m</i> -100 <i>m</i> records, 1 analogy query			
	<i>query</i> : select rows $a, b$ where $\langle k, v \rangle \in a$ , $\langle k, v' \rangle \in b$ , infer					b, infer $v$ from item memory and $v'$			
	benchmark	size parameters	benchmark sizes						
	set	k	1	2	3	4	5		
	db-match	(k,m)	(1,2)	(2,4)	(3,6)	(4,8)	(5,10)		
	kgraph	k	1	2	3	4	5		
	nfa	k	6	8	10	12	14		
	db-analogy	(k,m)	(4,1)	(8,2)	(12,3)	(16,4)	(20,5)		

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We parametrize	kgraph	threshold		1-100k edges/concept, 100k+10 concepts, 800k-1000k edges,				
			1 edge/query, 2 relations					
data structures	nfa	threshold	recognizes str with length $k$ , 1- $k$ character strings/query, 26 letters/alphabet					
with different		query: matc	hes are substrings of str, non-matches are partial substrings of str					
sizes	db-analogy	winner-take-all (WTA)	k/2-k fields/record, 50m-100m records, 1 analogy query					
51203.		<i>query</i> : select rows <i>a</i> , <i>b</i> where $\langle k, v \rangle \in a$ , $\langle k, v' \rangle \in b$ , infer <i>v</i> from item memory and <i>v'</i>						
1	benchmark	size parameters	benchmark sizes					
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We compared against dynamic tuning (**dt-all**) and other baselines, and configured all methods to deliver **99% query accuracy**.

#### **Optimization Runtime**

Heim runs in milliseconds, and is orders of magnitude faster than dynamic tuning-based approaches. (**2-5 orders of magnitude faster**)


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#### Accuracy

Heim-derived thresholds and sizes consistently meet accuracy target (above shaded region).



We sample 100 random data structures for each benchmark Y-axis is median accuracy, error bars are quartiles.

#### Accuracy

For several benchmarks, Heim is able to find parameterizations that dynamic tuning cannot find.



We sample 100 random data structures for each benchmark Y-axis is median accuracy, error bars are quartiles.

### **Optimal RRAM Density at Iso-Accuracy**

We can perform perform hardware-aware parameter optimization, and use the derived parameterizations to systematically analyze the tradeoffs between different device technologies / usages of device technologies.



1. Anjiang Wei, Akash Levy, Pu (Luke) Yi, Robert Radway, Priyanka Raina, Subhasish Mitra, and Sara Achour. 2023. PBA: Percentile-Based Level Allocation for Multiple-Bits-Per-Cell RRAM. In ICCAD. 76

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is Heim, is Heim-2bpc, is Heim-3bpc.

# In this case study, 2BPC ReRAM outperforms 3BPC ReRAM, 3BPC ReRAM worse than conventional memory at iso-accuracy across all hardware-optimized data structures. **Systematic applications-to-devices analysis!**

1. Anjiang Wei, Akash Levy, Pu (Luke) Yi, Robert Radway, Priyanka Raina, Subhasish Mitra, and Sara Achour. 2023. PBA: Percentile-Based Level Allocation for Multiple-Bits-Per-Cell RRAM. In ICCAD. 77

## Conclusion

We presented Heim, the first static analysis framework for hyper-dimensional computation that minimizes the resource usage in presence of hardware error

- Heim achieves better accuracy than dynamic tuning and offers guarantees, and is orders of magnitude faster
- Heim enables iso-accuracy systematic application-to-device analysis

We envision Heim as a sound core and basis for future analyses that may be unsound but apply to more practical applications